Methods of Estimating Methane Emissions from Albuquerque's Former Los Angeles Landfill

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Undergraduate Honors Thesis

Gabriella Dalton

University of New Mexico

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Abstract

Amidst the many sources of greenhouse gas emissions in the Unites States, primary landfills are an unsuspecting contributor to high levels of methane emissions exacerbating our global climate crisis. In fact, some estimates have predicted that primary landfills are among the three largest contributors to total methane emissions in the United States. [1] Methane is a common by-product of solid waste decomposition with an estimated Global Warming Potential (GWP) 28 times that of carbon dioxide. The amount of methane released from a given landfill source is highly variable as the stability and consistency of landfill environments are dynamic and dependent on multiple factors; nonetheless, it is important to accurately estimate methane emissions for record-keeping and environmental protection purposes.

In the United States, environmental scientists rely heavily on the U.S. Environmental Protection Agency?s (EPA) Landfill Gas Emissions Model (LandGEM) Version 3.02 to predict the generation of greenhouse gas levels present in landfills. This study explains the underlying mathematics of LandGEM, and uses empirical data from the Albuquerque Environmental Heath Department (AEHD) for the former Los Angeles landfill located off Paseo Del Norte in Albuquerque, New Mexico to assess the model.

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Chapter 1

Introduction

1.1 LandGEM

Landfills are one of the largest sources of greenhouse gas emissions in the United States and have some of the most difficult emission rates to quantify. The difficulties in accurately predicting these emission rates lie in the physical instabilities in moisture content, temperature and composition of landfill waste. Many of the methods used to quantify these emission rates are either overly simplified or inaccurate. This is due to a variety of uncertainties present in the decomposition of waste when moisture is a prominent factor. Unfortunately, current landfill gas models do not adequately account for the large variation in environmental conditions present in New Mexico.

In the United States, environmental scientists rely heavily on the U.S. Environmental Protection Agency's (EPA) Landfill Gas Emissions Model (LandGEM) Version 3.02 to predict the generation of greenhouse gas levels present in landfills. LandGEM is an automated estimation tool which utilizes a Microsoft Excel interface. The model is provided by the U.S. EPA and can be used to estimate emission rates for total landfill gas, methane, carbon dioxide, non-methane organic compounds, and individual air pollutants from municipal solid waste (MSW) landfills [2]. The model is based on a classic, first-order decay equation which is designed to evaluate annual gas emissions over a set period of time. The excel workbook used to estimate such emission rates takes on a relatively simple approach and provides a practical user interface for individuals in the landfill gas industry. Within this workbook, LandGEM provides model defaults, which quantify generalized emission predictions for a given landfill in the absence of site-specific data. The model defaults, published in 2008, were developed using empirical data from landfills around the United States. If these model defaults do not adequately represent emission characteristics at a given landfill, organizations using the workbook are able to upload site-specific data for their landfill to the screening tool. The screening tool allows users to input open and closure dates for the given landfill, as well as the weight capacity of the landfill. Since LandGEM is an automated tool, the U.S. EPA encourages individuals using the tool to take into account that the accuracy of LandGEM's predicted generation rates are dependent on the accuracy of their field data. Essentially, the more precise the input data, the more accurate the emission estimates. [2]

This research was conducted to determine the underlying mathematics of LandGEM while using empirical data from the former Los Angeles landfill to evaluate the accuracy of the model. The basic steps required in achieving this goal include understanding the connection between the amount of mass in the landfill and the flow rate of methane at a given time. This involved deriving an equation to represent the total amount of waste accepted at a specified time through the use of a source term. A source term is needed to represent the rate at which mass is added while mass is simultaneously undergoing anaerobic decomposition. This study will discuss two different source terms. Using this derived equation, the formula used in LandGEM is explained and the volume of methane produced by a landfill can be predicted. In the following section we will discuss the historical background of the former Los Angeles landfill and the underlying importance of accurately capturing greenhouse gas emissions from closed landfills. Section 2 provides the mathematical derivation of the LandGEM model, section 3 discusses the parameters in LandGEM while section 4 provides a best fit of the parameters to the Los Angeles landfill data. Finally, section 5 is a conclusion of our study.

1.2 Synopsis of the Former Los Angeles Landfill

1.2.1 Background of Historical Practices

The landfill was located two miles east of the Rio Grande River and one mile west of I-25, between Alameda Blvd on the north and Paseo del Norte Blvd on the south. [8] Before the landfill was available for public use it was primarily utilized as a harbor for commercial sand and gravel. During this time, the landfill was the only landfill in operation within the city limits of Albuquerque, New Mexico.The landfill first opened in 1978 and closed five years later in 1983. The northern region of the landfill was filled first and consisted of approximately 42 acres of the 77 intended for operational use. Due to the landfill's central location during the late 1970s, the 77 acres of land filled at a much quicker rate than operating officials had anticipated. Because the waste was not closely monitored, there poses the potential that among the solid waste lies hazardous waste. These waste particles can be in the form of liquids, solids, or gases that can be by-products of manufacturing processes, discarded used materials, or discarded unusual commercial products, such as cleaning fluids (solvents) or pesticides. [2]

In the summer of 1983, an environmental assessment study was performed on the northern region of the landfill. This assessment was conducted due to a forewarning that the generation of methane was present. As a result, an installation for an extraction system for landfill gas was suggested. In addition, in 1986, multiple groundwater monitoring wells were installed in order to avoid the potential risk of groundwater contamination. Approximately ten years after the first monitoring wells were installed, fifteen more monitoring wells had been installed along and within the perimeter of the landfill. All of which detected high concentrations of landfill gas and indicated that landfill gas was migrating outside the designated perimeters. As a result, more extraction wells and monitoring systems were installed within and outside of the specified regions of the landfill. The intention behind the installation of these systems was to curtail any, if not all, landfill gas present. Within the next year the extraction system was extended to the remaining regions of the landfill as a result of successful landfill gas removal from trial regions. The extraction system was effective for the remaining years that followed and in 2017 approximately six more wells were installed with the intention to increase recovery yield of methane throughout all regions of the landfill.

According to the 2017 Voluntary Abatement Plan, operation and Maintenance for the landfill gas extraction system involves routine and frequent adjustments to each individual extraction well. These adjustments prevent expulsion of methane into the environment and maintain at least a 30 percent by volume methane concentration. This amount is achieved to ensure the flare continues to burn and combust all extracted landfill gas without the input of supplemental fuel. The flare system currently operates continuously at an average yearly flow rate of 290 cfm, with the exception of shut downs during routine maintenance. As a result of this methane production, Interim Guidelines for Development within City of Albuquerque (City) Designated Landfill Buffer Zones are to be followed by all development within the city's jurisdiction. [3] These guidelines were put in place in order to protect the public and development that surrounds any given landfill. Due to the decomposition of buried waste, the generation of methane is unavoidable and is often a byproduct of decomposed waste with high levels of moisture present within the waste. The most common form of infiltration occurs through utility corridors, existing gravel and/or sand deposits below the surface or areas where prior excavations have occurred and the fill was not properly compacted. [3]Notably, the greater the generation of methane the greater the potential danger to those who reside or work near or in the landfill.

Figure 2.1 displays a schematijc illustration of the former Los Angeles Landfill and depicts the extraction system as it collects landfill gas from the waste and pulls it into the flare where it is burned along with other pollutants that have migrated out of the unlined landfill. [9]In addition to the description of the flare system, the schematic cross-section of the landfill also exhibits the groundwater treatment system that was installed in order to prevent groundwater contamination.

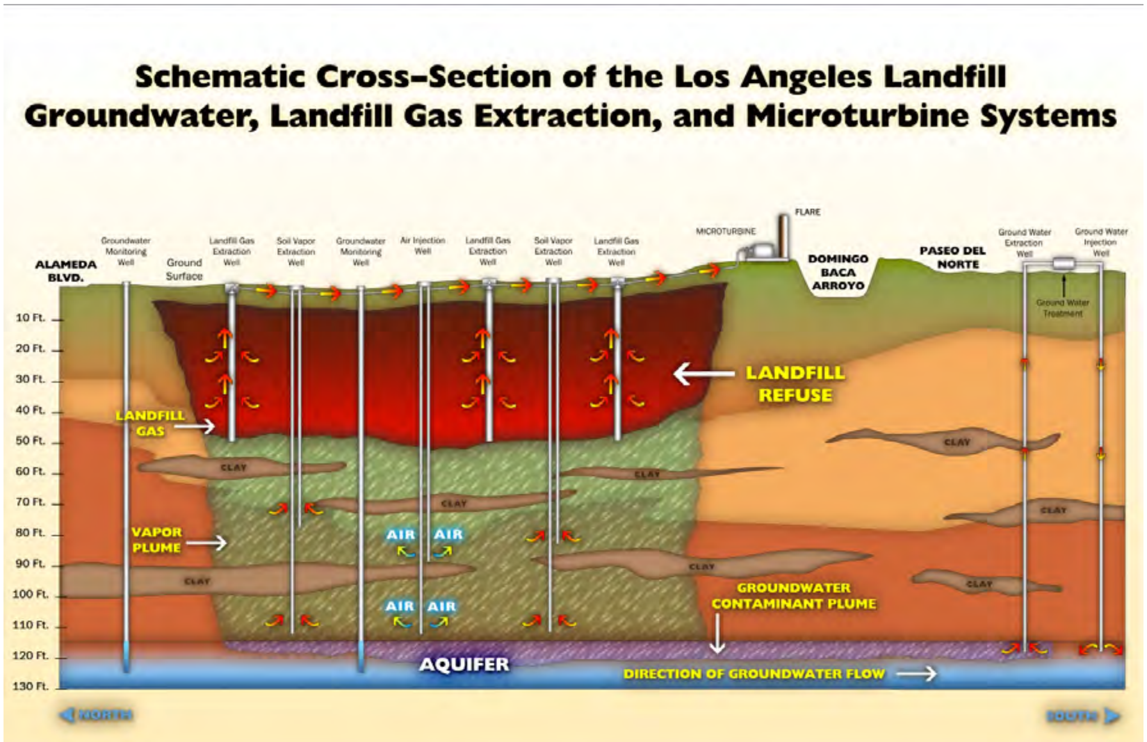


Figure 1.1: Schematic Cross-Section of the Los Angeles Landfill Groundwater, Landfill Gas Extraction, and Microturbine Systems

Chapter 2

LandGEM

2.1 Defining LandGEM

LandGEM is the most widely used landfill gas model in the United States, and is presented in the U.S. EPA guide as the following equation:

$$Q_{CH_4} = \sum_{i=1}^{n} \sum_{j=0.1}^{1} kL_0 \frac{M_i}{10} e^{-kt_{ij}} \tag{2.1}$$

where,
Q_{CH_4} = annual methane generation in the year of the calculation ($m^3/year$)
i = 1 year time increment
n= (year of the calculation) − (initial year of waste acceptance)
j = 0.1 year time increment
k= methane generation rate ($year^{-1}$)
L_0= potential methane generation capacity (m^3/Mg)
M_i= mass of waste accepted in the i^{th} year (Mg)
t_{ij} = age of the j^{th} section of waste mass M_i accepted in the i^{th} year (decimal years, e.g., 3.2 years)

In equation 2.1, the values of M_i would be site specific. There are two critical parameters k and L_0 in this model. The product of kL_0M_i has units of $\frac{m^3}{yr}$, so Q_{CH_4} would have the same units. At first sight, the meaning of t_{i_j} and the sum with j going from 0.1 to 1 is not clear. The origin of this equation for Q_{CH_4}, and even the meaning of Q_{CH_4} is not entirely obvious. The following sections attempt to understand and clarify equation (2.1).

2.2 LandGEM Model Analysis

Initially, the nature of LandGEM seemed to be somewhat elementary. The variables, constants, and parameters presented were comprehendible. Ultimately, the majority of trustworthy articles available have yet to properly describe LandGEM. Nearly all scholarly documents found relied solely on the LandGEM Version 3.02 User's Guide, as quoted in section (2.1), to explain the model. This guide states that LandGEM is derived from a simple-first order decomposition rate equation. References made to this generalized assumption include, "New and Improved Implementation of the First Order Model for Landfill Gas Generation or Collection" [5] and "Evaluation and Application of Site-Specific Data to Revise The First - Order Decay

Model for Estimating Landfill Gas Generation and Emissions at Danish Landfills"[7].

According to literature, a first order decay model has the form:

$$\frac{dM(t)}{dt} = -kM(t) \tag{2.2}$$

where, $M(t)$ represents the mass of waste at time, t, and the variable t denotes the time since the landfill was opened. The parameter, k is the decay rate of the mass. This differential equation describes the mass of the waste present in the landfill. The general solution to the above equation is:

$$M(t) = M_0 e^{-kt} \tag{2.3}$$

Initially, there would be no mass in the landfill. Though if $M_0 = 0$ then $M(t) = 0$ so something is missing. What is not taken into account is that mass is added to the landfill as the already placed waste undergoes anaerobic decomposition. As a result, the above equation 2.3 should be modified as follows:

$$\frac{dM(t)}{dt} = -kM(t) + \sigma(t), \qquad M(0) = 0 \tag{2.4}$$

where $\sigma(t)$ represents the rate at which mass is being added into the landfill. Some factors to take into account is that most commonly we only know the total mass added to a landfill. Sometimes we may know more about how the waste was added which can give us a better representation of the source term. However, we rarely know exact details about the waste such as monthly mass totals or varying waste composition types. For this study, we evaluated our first source term based on the assumption that all mass was dumped evenly throughout each year (rate: $\frac{M_i}{year}$). Our second source term was based on what turned out to be LandGEM's assumption that one tenth of the years mass was dumped instantaneously ten times during a year.

2.3 The Source Term, $\sigma(t)$

In general, to effectively analyze functions with jump discontinuities it is helpful to use functions known as unit step functions. The unit step function, or Heaviside function, is most commonly denoted by $H(t)$ and is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0. \end{cases} \tag{2.5}$$

To start, the source $\sigma(t)$ will be defined by

$$\sigma_1(t) = \sum_{i=1}^{n} M_i \left[H(t-i+1) - H(t-i)\right] \tag{2.6}$$

where, M_i is the mass of waste accepted in the i^{th} year (Mg). This source term assumes that we know the rate that waste is placed in the landfill for year i that the landfill is open. We assume in this model that this waste is placed in the landfill at a uniform rate. This rate should be represented as $\frac{M_i}{yr}$. The sum over $i = 1$ to n is a sum over the n year's that the landfill is open and accepting waste.

2.4 Derivation of $M_1(t)$ Using $\sigma_1(t)$

Let $M_1(t)$ denote the solution to (2.4) with source term given by (2.6). To solve for the mass of waste at a specific point in time, we first solve for $M_1(t)$. The solution can be obtained using Laplace transforms. Taking the Laplace transform of the source term yields

$$
\begin{aligned}
\mathcal{L}\{\sigma_1(t)\}(s) = \bar{\sigma}_1(s) &= \int_0^\infty \sigma_1(t)e^{-st}\mathrm{d}t \\
&= \sum_{i=1}^{n} \int_{i-1}^{i} M_i e^{-st}\mathrm{d}t \\
&= \sum_{i=1}^{n} \left[-\frac{M_i}{s} \left(e^{-si} - e^{-s(i-1)} \right) \right] \\
&= \sum_{i=1}^{n} \left[\frac{M_i}{s} \left(e^{-s(i-1)} - e^{-si} \right) \right]
\end{aligned} \tag{2.7}
$$

As we continue to compute the solution $M_1(t)$, transform (2.4):

$$
\begin{aligned}
\mathcal{L}[M_1'(t)](s) + k\mathcal{L}[M_1(t)] &= \sum_{i=1}^{n} \frac{M_i}{s}(e^{-s(i-1)} - e^{-si}) \\
s[\mathcal{L}M_1(t)](s) - M_1(0) + k\mathcal{L}[M_1(t)] &= \sum_{i=1}^{n} \frac{M_i}{s}(e^{-s(i-1)} - e^{-si}) \\
\mathcal{L}[M_1(t)](s) &= \sum_{i=1}^{n} \frac{M_i}{s(s+k)}(e^{-s(i-1)} - e^{-si})
\end{aligned} \tag{2.8}
$$

Using the method of Partial Fraction Decomposition we obtain,

$$
\frac{1}{s(s+k)} = \frac{k^{-1}}{s} - \frac{k^{-1}}{s+k}. \tag{2.9}
$$

Thus,

$$
\mathcal{L}[M_1(t)](s) = \sum_{i=1}^{n} \frac{M_i}{k} \left[e^{-s(i-1)} - e^{-si} \right] \left[\frac{1}{s} - \frac{1}{s+k} \right]. \tag{2.10}
$$

After applying the inverse Laplace transform, using the following formulas:

$$
\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[e^{at}] = \frac{1}{s-a}, \quad \mathcal{L}[H(t-c)f(t-c)] = e^{-cs}F(s) \tag{2.11}
$$

we obtain,

$$
M_1(t) = \sum_{i=1}^{n} \frac{M_i}{k} \left[H(t-i+1) - H(t-i) - H(t-i+1)e^{-k(t-i+1)} + H(t-i)e^{-k(t-i)} \right] \tag{2.12}
$$

Lastly, through further algebraic simplification, $M_1(t)$ becomes:

$$
M_1(t) = \sum_{i=1}^{n} \frac{M_i}{k} \left[\left(1 - e^{-k(t-i+1)} \right) H(t-i+1) - \left(1 - e^{-k(t-i)} \right) H(t-i) \right] \tag{2.13}
$$

2.5 Relationship Between $M(t)$ and Q_{CH_4}

In order to fully grasp the relationship linking $M(t)$ and Q_{CH_4} we must first grasp the connection between the two quantities. Essentially, Q_{CH_4} is the rate at which methane is produced. LandGEM assumes that if ΔM mass decays then $L_0\Delta M$ is the volume, ΔV, of methane produced. Thus, the rate at which methane is produced is represented as $\lim_{\Delta t \to 0} \frac{L_0 \Delta M}{\Delta t} = \frac{dV}{dt}$. In LandGEM, mass is measured in Mg and L_0 has units of $(\frac{m^3}{Mg})$. So, one Mg of mass has the potential to produce $L_0 m^3$ of methane.

Thus, after year n when $\sigma(t) = 0$ evaluate the following:

$$\Delta V = L_0 \Delta M \tag{2.14}$$

where $\Delta M = M(t) - M(t + \Delta t)$. Note that ΔM is positive since M(t) is decaying in time.
Thus,

$$\Delta V = L_0 \left[M(t) - M(t + \Delta t) \right]. \tag{2.15}$$

As we evaluate the change in the volume with respect to time,

$$\frac{\Delta V}{\Delta t} = L_0 \left[\frac{M(t) - M(t + \Delta t)}{\Delta t} \right]. \tag{2.16}$$

Now as we evaluate the limit as Δt approaches 0,

$$\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \lim_{\Delta t \to 0} \left[L_0 \left(\frac{M(t) - M(t + \Delta t)}{\Delta t} \right) \right] \tag{2.17}$$

.

which yields

$$\frac{dV}{dt} = -L_0 \frac{dM}{dt}. \tag{2.18}$$

Thus, after substituting $-kM$ for $\frac{dM}{dt}$,

$$Q_{CH_4} = \frac{dV}{dt} = kL_0 M. \tag{2.19}$$

Now, evaluate $\sigma(t) \neq 0$:

$$\Delta V = L_0 \Delta M \tag{2.20}$$

where $\Delta M = M(t) - M(t + \Delta t) + \sigma(t)\Delta t$.

Thus,

$$\Delta V = L_0 \left[M(t) - M(t + \Delta t) + \sigma(t)\Delta t \right] \tag{2.21}$$

As we evaluate the change in the volume with respect to time,

$$\frac{\Delta V}{\Delta t} = L_0 \left[\frac{M(t) - M(t+\Delta t)}{\Delta t} + \sigma(t) \right] \tag{2.22}$$

Now as we evaluate the limit as Δt approaches 0,

$$\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \lim_{\Delta t \to 0} L_0 \left[\frac{M(t) - M(t+\Delta t)}{\Delta t} + \sigma(t) \right] \tag{2.23}$$

yields

$$\frac{dV}{dt} = L_0 \left[-\frac{dM}{dt} + \sigma(t) \right]. \tag{2.24}$$

Thus, after substituting $-kM + \sigma(t)$ for $\frac{dM}{dt}$,

$$Q_{CH_4} = \frac{dV}{dt} = kL_0 M. \tag{2.25}$$

Hence, with or without the source term we obtain the same expression as in (2.25) for Q_{CH_4}. The units of Q_{CH_4} are $\frac{m^3}{yr}$.

2.6 Solution for $V_1(t)$

Now that $M_1(t)$ has been derived, consider the corresponding cumulative volume of methane release, $V_1(t)$.

$$V_1(t) = \int_0^t \frac{dV_1}{dt} = \int_0^t kL_0 M_1(t) \mathrm{d}t \tag{2.26}$$

Using the formula (2.13) integrate to get $V_1(t)$.

For each term in the sum, when $t < i$ the evaluated integral is zero.

For $i - 1 \leq t < i$

$$\begin{aligned} \int_{i-1}^t \left(1 - e^{-k(t-i+1)}\right) \mathrm{d}t &= \left[t + \frac{1}{k} e^{-k(t-i+1)} \right] \Big|_{i-1}^t \\ &= t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} \end{aligned} \tag{2.27}$$

For $t \geq i$

$$\begin{aligned} \int_{i-1}^t \left(1 - e^{-k(t-i+1)}\right) \mathrm{d}t - \int_i^t \left(1 - e^{-k(t-i)}\right) \mathrm{d}t &= \left[t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 \right] - \frac{1}{k} - \left[t + \frac{1}{k} e^{-k(t-i)} \right] \Big|_i^t \\ &= \frac{1}{k} e^{-k(t-i+1)} - \frac{1}{k} e^{-k(t-i)} + 1 \end{aligned} \tag{2.28}$$

Thus,

$$V_1(t) = M_i L_0 \sum_{i=1}^n \begin{cases} 0 & \text{if } t < i - 1, \\ t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} & \text{if } i - 1 \leq t < i, \\ \frac{1}{k} e^{-k(t-i+1)} - \frac{1}{k} e^{-k(t-i)} + 1 & \text{if } t \geq i. \end{cases} \tag{2.29}$$

Representing $V_1(t)$ by unit step functions yields the following:

$$V_1(t) = M_i L_0 \sum_{i=1}^{n} \left[\left(t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} \right) (H(t-i+1) - H(t-i)) \right] + \left[\left(\frac{1}{k} e^{-k(t-i+1)} - \frac{1}{k} e^{-k(t-i)} + 1 \right) H(t-i) \right] \tag{2.30}$$

Hence, the final representation of $V_1(t)$ is as follows:

$$V_1(t) = M_i L_0 \sum_{i=1}^{n} \left[\left(t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} \right) H(t-i+1) - \left(\frac{1}{k} e^{-k(t-i)} + t - i - \frac{1}{k} \right) H(t-i) \right] \tag{2.31}$$

$V_1(t)$ is the cumulative amount of methane produced by the landfill up to time, t with the given initial conditions $V_1(0) = 0$.

2.6.1 Verifying $V_1(t)$

To check that this formula (2.31) makes sense, look at $V_1(t)$ and evaluate the limit as time, t, approaches ∞.

$$\begin{aligned}
\lim_{t \to \infty} V_1(t) &= \lim_{t \to \infty} \left\{ L_0 \sum_{i=1}^{n} M_i \left[\left(t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} \right) H(t-i+1) - \left(\frac{1}{k} e^{-k(t-i)} + t - i - \frac{1}{k} \right) H(t-i) \right] \right\} \\
&= L_0 \sum_{i=1}^{n} M_i \lim_{t \to \infty} \left[t + \frac{1}{k} e^{-k(t-i+1)} - i + 1 - \frac{1}{k} - \frac{1}{k} e^{-k(t-i)} - t + i + \frac{1}{k} \right] \\
&= L_0 \sum_{i=1}^{n} M_i
\end{aligned} \tag{2.32}$$

The sum $\sum_{i=1}^{n} M_i$ is the total mass placed in the landfill while it is open. Eventually all the mass would decay and produce $L_0 \sum_{i=1}^{n} M_i$ methane.

2.6.2 Plot of $V_1(t)$

Figure 2.1 displays a graph of the amount of methane present in a landfill as a function of time, t. In order to check the validity of our equation for the cumulative amount of methane, $V_1(t)$, we numerically integrated using a cumulative trapezoidal numerical integration method in MATLAB. The blue curve labeled "$V1(t)$ Fit" denotes the numeral fit using the built-in MATLAB command "cumtrapz"; the orange curve labeled "V1(t)" is evaluated using (2.31).

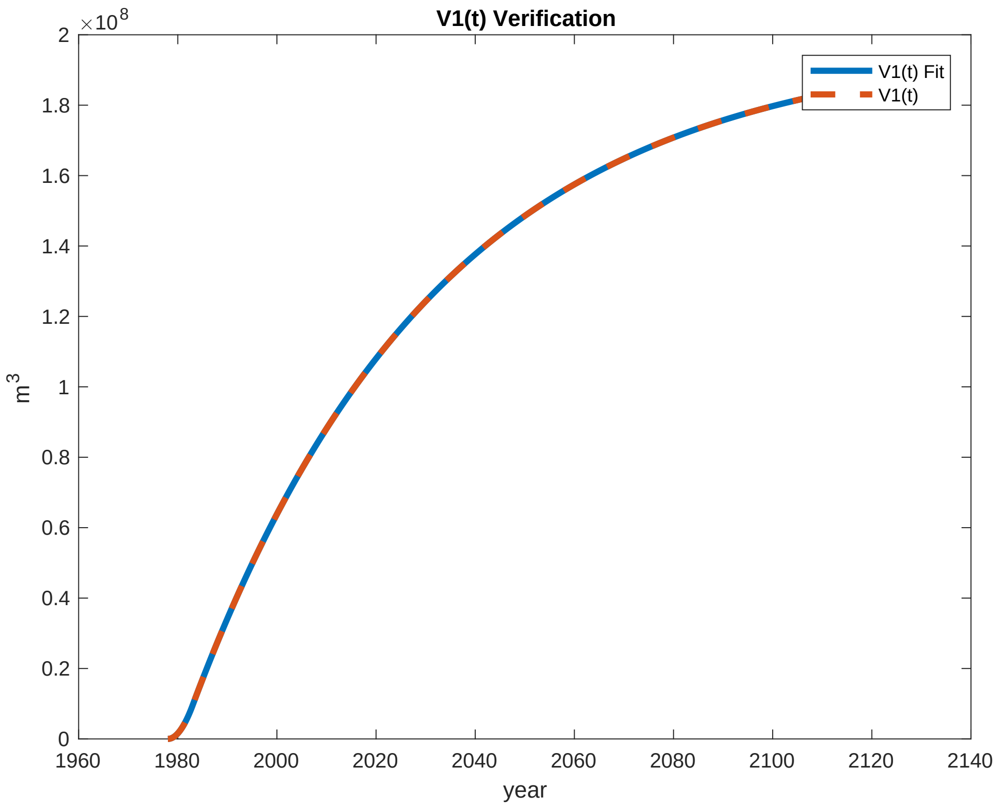


Figure 2.1: $V_1(t)$ vs. time, t

2.7 Derivation of $M_2(t)$

Now, we will solve for $M_2(t)$ using equation 2.5 and

$$\sigma_2(t) = \sum_{i=1}^{n}\sum_{j=1}^{10} \frac{M_i}{10}\delta\left(t-(i-1)-\frac{j}{10}\right) \tag{2.33}$$

where $\sigma_2(t)$ is our source term which denotes the rate which mass is being added into the landfill. The delta function models dumping one tenth of the year's mass. M_i each tenth of a year. It will turn out that this source term is what is assumed in LandGEM to give the formula for Q_{CH_4} in section 2.1.
Taking the Laplace Transform of the right-hand side of the function yields to the following representation:

$$\begin{aligned}\mathcal{L}\{\sigma_2(t)\}(s) = \bar{\sigma_2}(s) &= \int_0^\infty \sum_{i=1}^{n}\sum_{j=1}^{10} \frac{M_i}{10}\delta\left(t-(i-1)-\frac{j}{10}\right)e^{-st}\mathrm{d}t \\ &= \sum_{i=1}^{n}\sum_{j=1}^{10} \frac{M_i}{10}\int_0^\infty \delta\left(t-(i-1)-\frac{j}{10}\right)e^{-st}\mathrm{d}t\end{aligned} \tag{2.34}$$

As we continue to compute $M_2(t)$ using Laplace transforms.

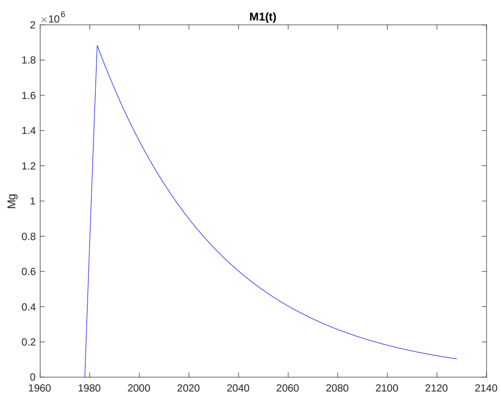
$$
\begin{aligned}
\mathcal{L}[\sigma_2(t)] &= \sum_{i=1}^{n}\sum_{j=1}^{10}\frac{M_i}{10}e^{-s(i-1+\frac{j}{10})} \\
\mathcal{L}[M_2'(t)](s) + k\mathcal{L}[M_2(t)] &= \sum_{i=1}^{n}\sum_{j=1}^{10}\frac{M_i}{10}e^{-s(i-1+\frac{j}{10})} \\
s\mathcal{L}[M_2(t)](s) - M_2(0) + kL[M_2(t)] &= \sum_{i=1}^{n}\sum_{j=1}^{10}\frac{M_i}{10}e^{-s(i-1+\frac{j}{10})} \\
\mathcal{L}M_2(t)](s) &= \sum_{i=1}^{n}\sum_{j=1}^{10}\frac{M_i}{10}e^{-s\left(i-1+\frac{j}{10}\right)}\left(\frac{1}{s+k}\right)
\end{aligned} \tag{2.35}
$$

Thus, the final solution for $M_2(t)$ is

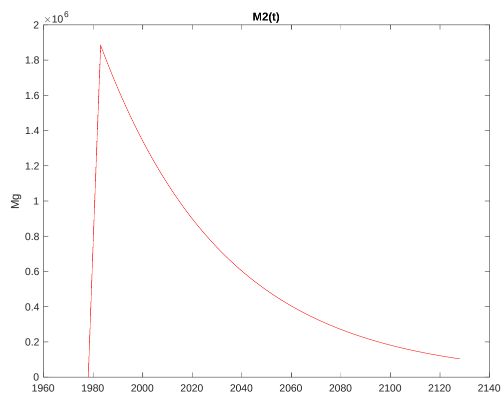
$$
M_2(t) = \sum_{i=1}^{n}\sum_{j=1}^{10}\frac{M_i}{10}e^{-k\left(t-(i-1)-\frac{j}{10}\right)}H\left(t-(i-1)-\frac{j}{10}\right) \tag{2.36}
$$

2.8 Plot of $M_1(t)$ and $M_2(t)$

Using Albuquerque's Los Angeles landfill as a guide, use $n = 5$, since the landfill was in operation from 1978-1983. We also know that approximately a total of 1,978,700 Mg of waste was put in the landfill during those 5 years. Assuming equal amounts were deposited each year, set $M_i = m = 395,740$ Mg for $i = 1, 2, ..., n$. We use k=0.02 based on the U.S. EPA's estimate of the decay rate for an arid environment [2].



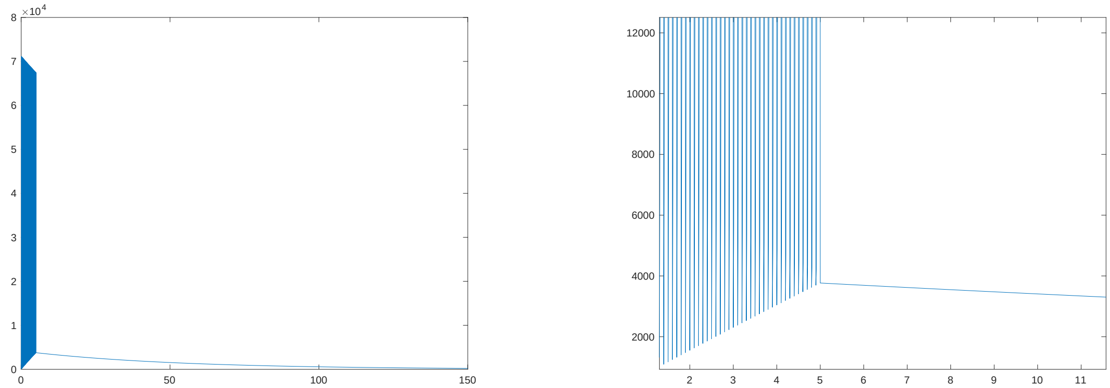
(a) $M_1(t)$



(b) $M_2(t)$

Figure 2.2: Plot of $M_1(t)$ and $M_2(t)$

There is a small difference in the predictions based on the two source terms, (2.6) and (2.33). Plotted in Figure 2.3 is a graph of these differences. There is a maximum ten percent difference during the years of operation when the source term differs, but shortly after the landfill closes there is very little difference between the models.



(a) Difference Between $M_1(t)$ and $M_2(t)$

(b) A Zoomed In Difference Between $M_1(t)$ and $M_2(t)$

Figure 2.3: Comparison of Model Differences Between $M_1(t)$ and $M_2(t)$

2.9 Derivation of $V_2(t)$

As in section 2.7, we can define the cumulative methane release by

$$V_2(t) = \int_0^t \frac{dV_2}{dt} = \int_0^t kL_0 M_2(t)\mathrm{d}t. \tag{2.37}$$

Substituting the expression (2.36) for $M_2(t)$ and integrating yields $V_2(t)$.

For each term in the sum, when $t < i - 1 + \frac{j}{10}$ the evaluated integral is zero.

For $t \geq i - 1 + \frac{j}{10}$

$$\int_{i-1+\frac{j}{10}}^{t} \left[e^{-k\left(t-(i-1)-\frac{j}{10}\right)}\mathrm{d}t\right] = -\frac{1}{k}e^{-k\left(t-(i-1)-\frac{j}{10}\right)} + \frac{1}{k} \tag{2.38}$$

Thus,

$$V_2(t) = \frac{L_0}{10}\sum_{i=1}^{n}\sum_{j=1}^{10} M_i \begin{cases} 0 & \text{if } t < i-1+\frac{j}{10}, \\ 1 - e^{-k\left(t-(i-1)-\frac{j}{10}\right)} & \text{if } t \geq i-1+\frac{j}{10}. \end{cases} \tag{2.39}$$

Finally, representing $V_2(t)$ through unit step functions is obtained through the following:

$$V_2(t) = \frac{L_0}{10}\sum_{i=1}^{n}\sum_{j=1}^{10} M_i \left(1 - e^{-k\left(t-(i-1)-\frac{j}{10}\right)}\right) H\left(t-(i-1)-\frac{j}{10}\right) \tag{2.40}$$

2.9.1 Verifying $V_2(t)$

To clarify that $V_2(t)$ is the correct solution to the evaluated integral above we are able to evaluate the limit as time, t approaches ∞.

$$\begin{aligned}
\lim_{t\to\infty} V_2(t) &= \frac{L_0}{10} \lim_{t\to\infty} \left\{ \sum_{i=1}^{n} \sum_{j=1}^{10} M_i \left(1 - e^{-k\left(t-(i-1)-\frac{j}{10}\right)}\right) H\left(t-(i-1)-\frac{j}{10}\right) \right\} \\
&= \frac{L_0}{10} \sum_{i=1}^{n} \sum_{j=1}^{10} M_i \\
&= \frac{L_0}{10} \sum_{i=1}^{n} 10 M_i \\
&= L_0 \sum_{i=1}^{n} M_i .
\end{aligned} \tag{2.41}$$

As discussed for the previous model, this is the correct limiting value for $V_2(t)$.

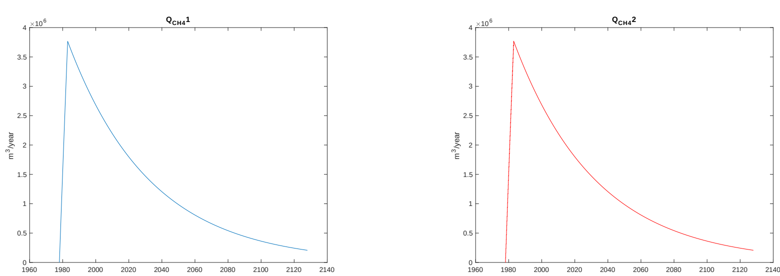
2.10 Plot of Q_{CH_4}

Plotted below is the flow rate of methane ($\frac{m^3}{year}$) vs. time (year), which was developed through MATLAB using equations (2.13) and (2.36). The same parameters are used in these plots as for Figure (2.1), with the addition of $L_0 = 100$ ($\frac{m^3}{Mg}$) as recommended by the U.S. EPA for an arid environment. Note that $\frac{dV}{dt}$ (i.e. Q_{CH_4}) is $kL_0M(t)$. This provides us with the mathematical relationship between Q_{CH_4} and $M(t)$.

It is now clear that Q_{CH_4} should be a function of t, $Q_{CH_4}(t) = \frac{dV}{dt} = kL_0M(t)$. Using the source term $\sigma_2(t)$ leads to the formula $M_2(t)$ for the mass in the landfill at time, t. Thus, the emission rate of methane with this source term is (equation 2.42). This formula is now a precise mathematical formula for Q_{CH_4} in section 2.1.

$$Q_{CH_4} = \frac{dV_2}{dt} \quad = \sum_{i=1}^{n} \sum_{j=1}^{10} \frac{M_i}{10} k L_0 \left[e^{-k\left(t-(i-1)-\frac{j}{10}\right)} H\left(t-(i-1)-\frac{j}{10}\right) \right]. \tag{2.42}$$

2.10.1 Plot of Q_{CH_4} Using (2.13) and (2.36)



(a) Q_{CH_4} using (2.13)

(b) Q_{CH_4} using (2.36)

Figure 2.4: Plot of Q_{CH_4} using $M_1(t)$ and $M_2(t)$

Chapter 3

LandGEM Worksheets

LandGEM Version 3.02 uses nine worksheets located in a Microsoft Excel Workbook to analyze data from either selected default parameters or site-specific data. The “Introduction” worksheet consists of a generalized overview of methods utilized by LandGEM and highlights key concepts. The “User Inputs” worksheet allows users to input their individualized waste characteristics, including waste acceptance rates, model parameters, and pollutant/gas compositions. The “Pollutants” worksheet requires pollutant concentration inputs, and considers the molecular weight of all pollutants in order to determine which pollutants will be dispersed into the atmosphere and which will settle in the landfill. The “Input Review” worksheet provides users with an opportunity to review all previously input data. The “Methane” worksheet uses a first-order decay equation to estimate for all time t, the rate of methane generation. The “Results” worksheet displays an overview of calculations conducted using data from the ”User Inputs” worksheet. The “Graphs” worksheet allows users to visualize emission data calculated by LandGEM. The “Inventory” worksheet collects estimated emission figures for each pollutant/gas described in the “Pollutants” worksheet over a year time span. Lastly, the “Report” worksheet provides a summary of calculations and figures obtained in previous worksheets.

3.1 Understanding Parameters of LandGEM

LandGEM depends on two critical parameters to determine landfill gas emission rates. These parameters represent the potential methane generation capacity (L_0) and the generation rate of methane (k). Default values are located under the user input section in the Microsoft Excel Workbook. When using LandGEM’s default values, users must use the same values for the two critical model parameters used to quantify the rate of emissions accurately. For example, if “Inventory Arid Area - 0.02 ” is chosen for k then “Inventory Arid Area - 100” must be chosen for L_0 as well. The same process should be followed when choosing the preceding default values. These parameter values are based on data provided by the U.S. EPA. Consequently, when entering site-specific data users are able to enter differing parameter values from the default values to accurately represent the landfill of their choice.

3.1.1 Potential Methane Generation Capacity, L_0

The Potential Methane Generation Capacity, L_0, depends on waste type and characteristic makeup of the waste present in the landfill. This potential is based on the rate of degradable organic material in the landfill. In essence, the value of L_0 increases as the amount of organic matter in a landfill increases. As this material accumulates, the potential generation of methane in the landfill increases as the waste undergoes anaerobic decomposition. Anaerobic decomposition occurs when decomposing matter is not exposed to oxygen, and in turn causes methane generation. To determine the value of L_0, LandGEM offers five default values based on emission and landfill type. These parameter values are:

CAA	Conventional	170
CAA	Arid Area	170
Inventory	Conventional	100
Inventory	Arid Area	100
Inventory	Wet (Bioreactor)	96

"The CAA defaults are based on federal regulations for MSW landfills laid out by the Clean Air Act (CAA)." [2]

3.1.2 Methane Generation Rate, k

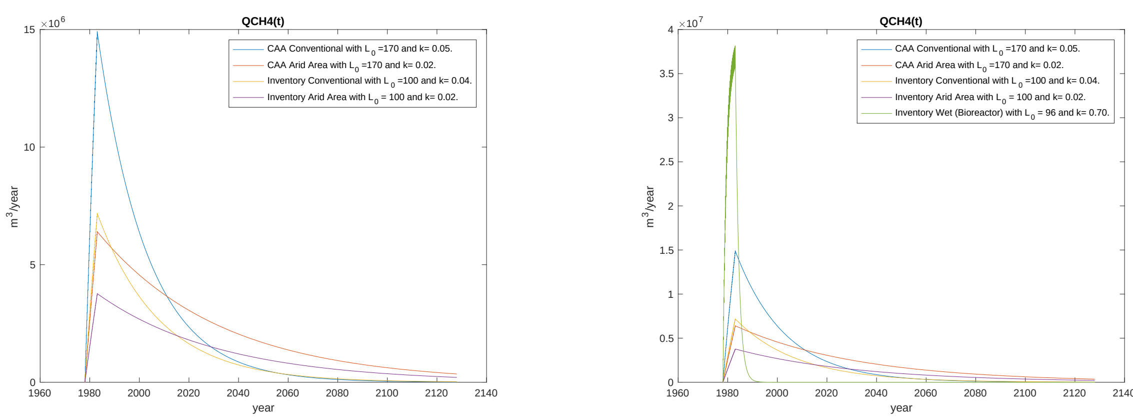
The Methane Generation Rate, k, determines the rate of decomposition of the waste's mass. The rate is dependent on moisture content, waste partial size, pH, and waste composition. Moisture content within a landfill depends on the initial moisture content of the waste, operational practices and rainfall that permeates the landfill's surface.[6] LandGEM's default k values are based on default type (CAA and Inventory) and landfill type to determine the appropriate value of k. These parameters values consist of the following:

CAA	Conventional	0.05
CAA	Arid Area	0.02
Inventory	Conventional	0.04
Inventory	Arid Area	0.02
Inventory	Wet (Bioreactor)	0.70

These variations in the k value are primarily dependent on the increase of moisture content, which in turn independently increases the amount of methane produced.

3.2 Plot of Default Parameters, k and L_0

Below is a graph of the flow rate of methane, $Q_{CH_4}(t)$, for the default parameters discussed in sections (3.1.1) and (3.1.2) as recommended by the U.S. EPA for various landfill conditions.



(a) $Q_{CH_4}(t)$ in $\frac{m^3}{yr}$ When Bioreactors Are Not Present Using Default Parameters in (3.1.1) and (3.1.2)

(b) $Q_{CH_4}(t)$ in $\frac{m^3}{yr}$ When Bioreactors Are Present Using Default Parameters in (3.1.1) and (3.1.2)

Figure 3.1: Graph of $Q_{CH_4}(t)$ Based on Varying Parameter Values

Chapter 4

Analysis of Empirical Data Provided by the AEHD

4.1 Background on the Data

During the summer and fall semester of 2017 in the course of my final year as an undergraduate student, I worked alongside professional engineers and scientists at AEHD as their apprentice. Throughout my time at the AEHD, my primary duty was to study the trends in landfill gas perturbations with an emphasis on rising methane emission levels from closed landfills. One of my main responsibilities was to dissect the ins and outs of the U.S. EPA's model, LandGEM. In order to fully understand the model, I was provided an empirical dataset from my colleagues which consisted of measured flow rates from February 2001 until March 2017. These flow rates were measured by the landfill gas extraction system, as well as the flare. These two methods of landfill gas monitoring provide source control for landfill gas emanating from the landfill within the body of the remaining landfill waste. [4] Unfortunately, due to the fact that elevated methane levels were not detected until June 1995 and electronic data were not provided until February 2001, there exists an eighteen year gap in the data provided by the AEHD from when the landfill closed until data collection began.

The excel workbook that contains the data consists of measurement dates, the method used to capture the landfill gas, percentage of methane present during the time of extraction, cfm rates, and lastly, measurements for the cumulative flow rate of methane in megagrams per year. Due to the fact that the flow rates were measured in megagrams per year instead of cubic meters per year, a conversion method was used to convert the data to the proper units.

4.1.1 Converting Data from $\frac{Mg}{yr}$ to $\frac{m^3}{yr}$:

In order to begin converting the data, we first need to know that the molar mass of methane is considered to be approximately 16.04 $\frac{g}{mol}$ and at STP 1 mole of gas occupies 22.4L .

Consequently,

$$\left[\frac{Mg}{yr}\right]\left[\frac{10^6 g}{Mg}\right]\left[\frac{mol}{16.04g}\right]\left[\frac{22.4L}{mol}\right]\left[\frac{10^{-3}m^3}{1L}\right] = 1396.31\frac{m^3}{yr}. \tag{4.1}$$

4.2 Verifying $Q_{CH_4}(t)$

Clearly, $Q_{CH_4}(t)$ is a scaled version of $M(t)$ since $Q_{CH_4}(t) = kL_0M(t)$. The mass $M(t)$ decays exponentially once there is no new mass being placed in the landfill. Thus, we expect $M(n)$ to be the maximum mass in the landfill. Similarly, $Q_{CH_4}(t_n)$ should be the maximum emission rate, with $Q_{CH_4}(t_n)$ decaying exponentially at subsequent times. Here, t_n is the year corresponding to the n^{th} year of operation, $t_n = t_0 + n$, where t_0 is the year the landfill opened. Therefore, we expect the portion of the curve $Q_{CH_4}(t)$ for $t \geq t_n$ to have the form

$$Q_{CH_4}(t) = Q_{CH_4}(t_n)e^{-kt}. \tag{4.2}$$

To estimate k from the data, take the log of equation (4.2)

$$log\,(Q_{CH_4}(t)) = log\,(Q_{CH_4}(t_n)) - kt. \tag{4.3}$$

Given data $(t_i, log\,(Q_{CH_4}(t_i)))$ at various times t_i, we want to find the least squares fit to k. Using MATLAB's polyfit routine we find $log\,(Q_{CH_4}(t_n))$ to be equal to 103.2351 and k to be equal to 0.0442, thus

$$log\,(Q_{CH_4}(t)) = 103.2351 - 0.0442t. \tag{4.4}$$

If we then take the exponential of both hand-sides of equation (4.4) we obtain:

$$\begin{aligned} Q_{CH_4}(t) &= e^{103.2351-0.0442t} \\ &= [e^{103.2351}][e^{-0.0442t}]. \end{aligned} \tag{4.5}$$

Thus, the value of k that best fits the data is $k = 0.0442$. Now, we would like to estimate L_0. Since $Q_{CH_4}(t_n) = kL_0M_2(n)$ we can solve for L_0 explicitly

$$L_0 = \frac{Q_{CH_4}(t_n)}{kM_2(n)}. \tag{4.6}$$

Using equation (4.8) we are able to solve for $Q_{CH_4}(t_n)$, where $t_n = 1983$, the year the landfill closed.

$$\begin{aligned} Q_{CH_4}(1983) &= e^{103.2351-0.0442(1983)} \\ &\approx 6.4273 \times 10^6. \end{aligned} \tag{4.7}$$

Note that our formula for $M_2(t)$ assumes that the landfill opens at $t = 0$ and closes at $t = n$. In order to find an approximation to the maximum mass in the landfill, we must first solve for $M_2(n)$ using equation(2.36). We note that for the Los Angeles landfill, $M_i = m$ for $i = 1, 2, .., n$ where m is a constant.

Thus,

$$
\begin{aligned}
M_2(n) &= \sum_{i=1}^{n}\sum_{j=1}^{10} \frac{M_i}{10}\left(e^{-k\left(n-(i-1)-\frac{j}{10}\right)}\right) H\left(n-(i-1)-\frac{j}{10}\right) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{10} \frac{m}{10}\left(e^{-k\left(n-(i-1)-\frac{j}{10}\right)}\right) \\
&= \frac{m}{10}e^{-kn}\sum_{i=1}^{n}\sum_{j=1}^{10} e^{k(i-1)}e^{\frac{kj}{10}} \\
&= \frac{m}{10}e^{-kn}\sum_{i=1}^{n} e^{k(i-1)}\sum_{j=1}^{10} e^{\frac{kj}{10}} \\
&= \frac{m}{10}e^{-kn}\sum_{i=1}^{n} e^{k(i-1)}\left(\frac{e^{\frac{k}{10}}(1-e^{k})}{1-e^{\frac{k}{10}}}\right) \\
&= \frac{m}{10}e^{-kn}\left(\frac{e^{\frac{k}{10}}(1-e^{k})}{1-e^{\frac{k}{10}}}\right)\sum_{i=1}^{n} e^{k(i-1)} \\
&= \frac{m}{10}e^{-kn}\left(\frac{e^{\frac{k}{10}}(1-e^{k})}{1-e^{\frac{k}{10}}}\right)\left(\frac{e^{kn}-1}{e^{k}-1}\right) \\
&= -\frac{m}{10}e^{-kn}\left(\frac{e^{\frac{k}{10}}}{1-e^{\frac{k}{10}}}\right)\left(e^{kn}-1\right)
\end{aligned} \tag{4.8}
$$

Hence, the exact value for $M_2(n)$ is:

$$
M_2(n) = \frac{m}{10}\left(\frac{e^{\frac{k}{10}}}{1-e^{\frac{k}{10}}}\right)\left(1-e^{-kn}\right). \tag{4.9}
$$

Now, since we are estimating $M_2(n)$ when $n = 5$, $m = 395,740$, $k = 0.0442$, and $kn = 0.2210$, $M_2(n)$ becomes

$$
M_2(5) \approx 1.7792 \times 10^6. \tag{4.10}
$$

Thus, we can solve for L_0:

$$
L_0 \approx \frac{6.4273 \times 10^6}{(1.7792 \times 10^6)(0.0442)} \quad \approx 81.7300. \tag{4.11}
$$

4.2.1 Plot of Polynomial Fit

Below is a graph of the empirical data provided by AEHD plotted against LandGEM. The LandGEM plotted is based on the critical parameters $k = 0.02$ and $L_0 = 100$, which best represent New Mexico's arid climate as recommended by the U.S. EPA. The goal of this assessment is to fit parameters k and L_0 to the AEHD data. In doing this, we found that the best fit values to represent the former Los Angeles landfill are $k = 0.0442$ and $L_0 = 81.7300$.

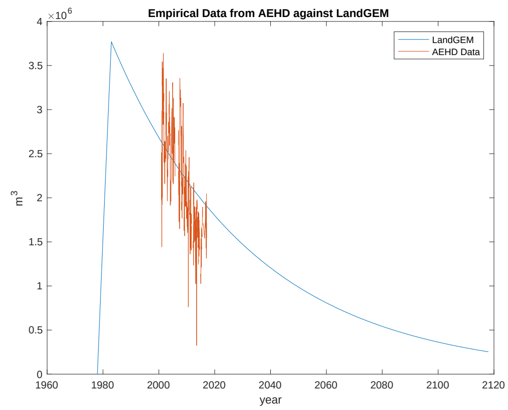
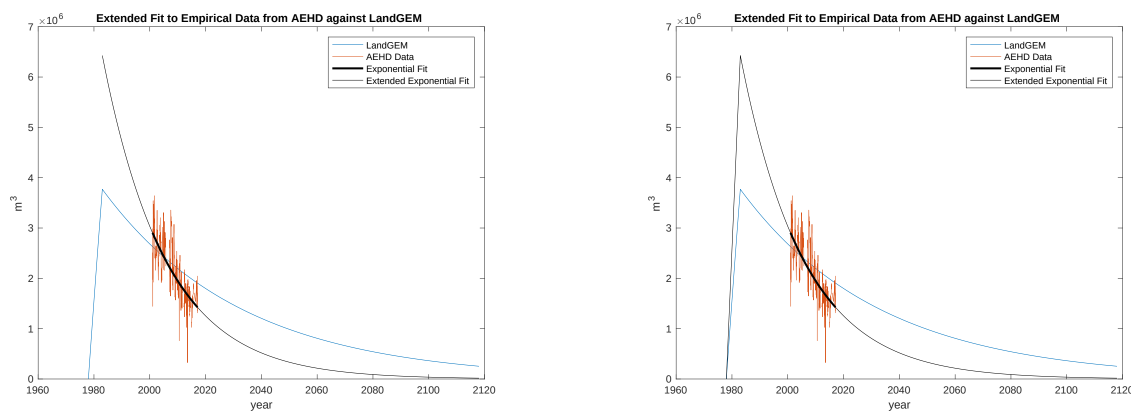


Figure 4.1: Empirical Data from AEHD against LandGEM with $k = 0.02$, $L_0 = 100$.



(a) Extended Fit to Empirical Data from AEHD against LandGEM with $k = 0.0442$

(b) Extended Fit to Empirical Data from AEHD against LandGEM with $k = 0.0442$, $L_0 = 81.7300$.

Figure 4.2: Best Fit Data For The Los Angeles Landfill

According to the U.S. EPA defaults, the value of k that best fits the data corresponds to a value more typical of a standard landfill rather than one in an arid environment. The value of L_0 is also more typical of wetter waste. Since the measured data are limited, it is not clear if these parameters also describe the landfill emissions during initial years of operation. It is possible that more recent housing development in the area surrounding the landfill has led to more water runoff infiltrating the landfill and increasing the emission rates.

Chapter 5

Conclusion

5.1 Summary

This study discussed the underlying mathematics of LandGEM. By formulating a model for the mass in the landfill at time, t that included a source term, we were able to illustrate how the U.S. EPA developed their model for estimating landfill gas. One of the primary focuses was to observe the relationship between the mass in the landfill at time, t, and the flow rate of methane. Subsequently, we were able to assess the model based on empirical data, which allowed us to fit the two critical parameters to the Los Angeles landfill using data provided by the AEHD.

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